$f(u) = au^3 + bu^2 + cu + d$

each row you can solve by constant

$U = [u^3 \quad u^2 \quad u \quad 1]$

$P = [p_0 \quad p_1 \quad p_2 \quad p_3]^T = \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$

for each u we use one p

$f(u) = U M P$

$au^3 + bu^2 + cu + d = [u^3 \quad u^2 \quad u \quad 1] \; M \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$

$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} [u^3 * u^2 * u \quad 1] = [u^3 \quad u^2 \quad u \quad 1] \; M \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$

$Ax = b$

$x = A^{-1} b$

$= p_0 + b p_1 + c p_2 +$

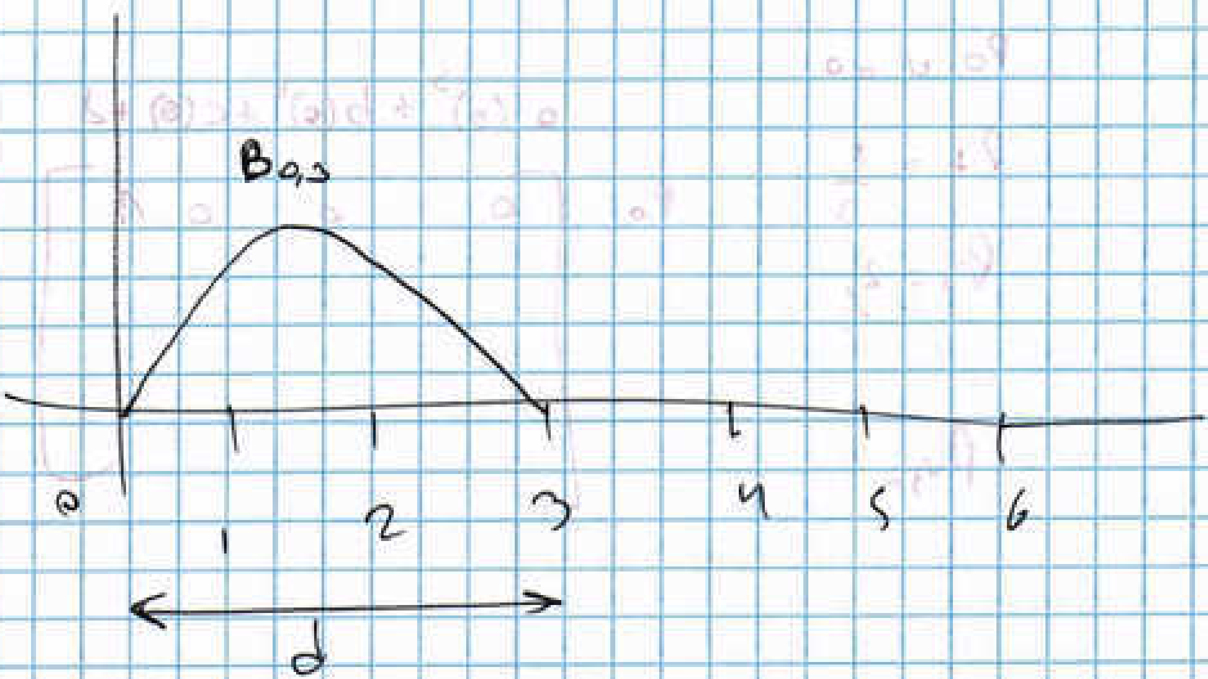
$P U M$

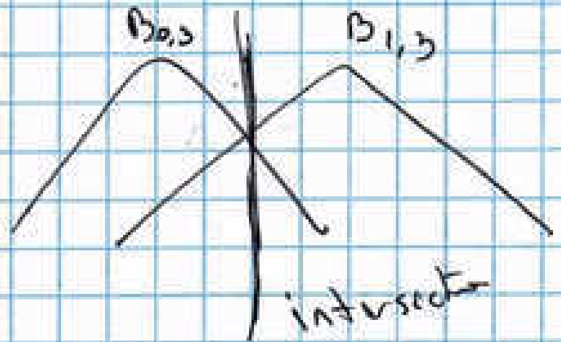
$Ax = f(u)$

$M = f(u) \, U^{-1} \, P^{-1}$

4

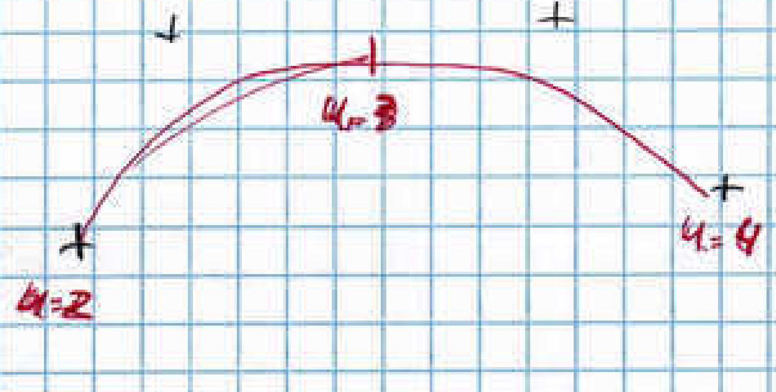
$\begin{array}{l} p_0 \\ 4 \; p_1 \\ p_2 \\ p_3 \end{array} \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$





start on $B_{0,3}$
end on $B_{1,3}$

control p_0 and p_1



194

$P_0 \quad u = 0$

$P_1 = \frac{1}{3}$

$P_3 = \frac{2}{3}$

$P_{y_u} =$

$a(0)^3 + b(0)^2 + c(0) + d$

$P_0 \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$

Part 1 :

$$f(u) = au^3 + bu^2 + cu + d$$

1)

$P_0 \to u = 0 \qquad f(0) = a(0)^3 + b(0)^2 + c(0) + d = a0 + b0 + c0 + d$

$P_1 \to u = 1/3 \qquad f(1/3) = a(1/3)^3 + b(1/3)^2 + c(1/3) + d = a\,1/27 + b\,1/9 + c\,1/3 + d$

$P_2 \to u = 2/3 \qquad f(2/3) = a(2/3)^3 + b(2/3)^2 + c(2/3) + d = a(8/27) + b(4/9) + c\,2/3 + d$

$P_3 \to u = 1 \qquad f(1) = a(1)^3 + b(1)^2 + c(1) + d = a + b + c + d$

$$\begin{array}{c} P_0 \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1/27 & 1/9 & 1/3 & 1 \\ 8/27 & 4/9 & 2/3 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1}$$

19(2) a. 6 control points

b. $k = \{0, 1, 2, 3, 4, 5\}$

c. $n+1 = 6 \therefore \boxed{n = 5}$

d. $d - 1 = 2$ (2nd degree) $\therefore d = 2+1 = 3 \therefore \boxed{d = 3}$

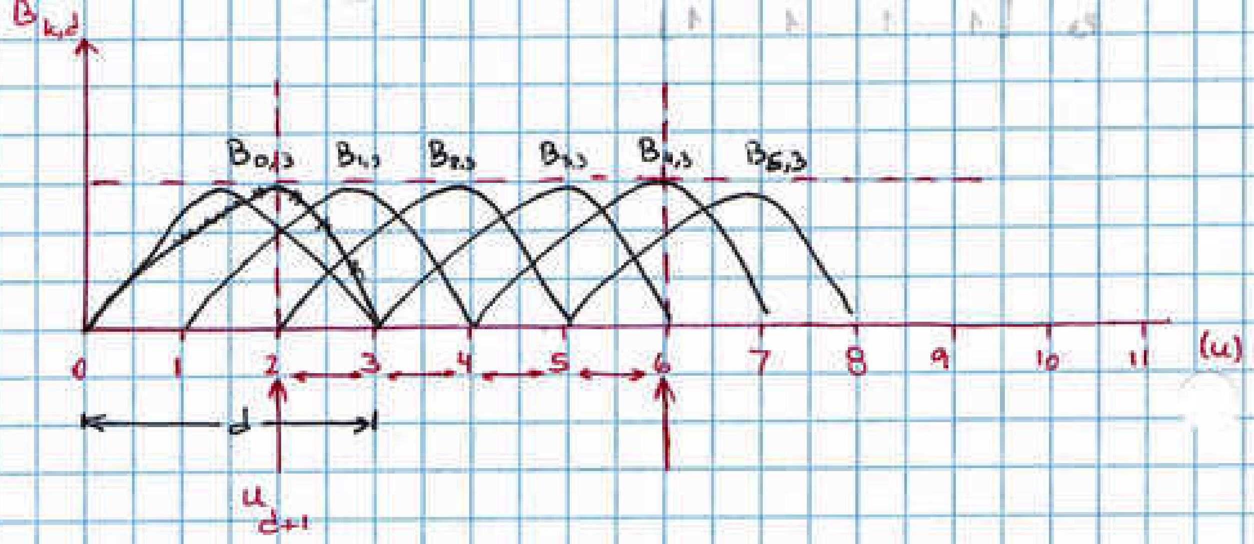
e. ~~#of~~ Number of knot values $= n + d + 1$

$= 5 + 3 + 1 = \boxed{9}$

$\{0, 1, 2, 3, 4, 5, 6, 7, 8\} = \{u_0, u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$

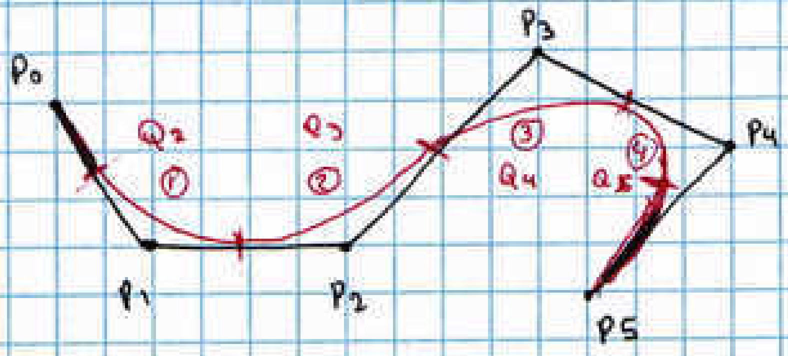
f. Division of intervals: $n + d = 5 + 3 = 8 \therefore u$ is divided in 8 uniform intervals

e.



start: $u_{d-1} = u_{3-1} = u_2$

end: $u_{n+1} = u_{5+1} = u_6$



Part 1

3) Number of Control Points: 8

$k = \{0, 1, 2, 3, 4, 5, 6, 7\}$

If $n+1 = 8$ then $\boxed{n = 7}$

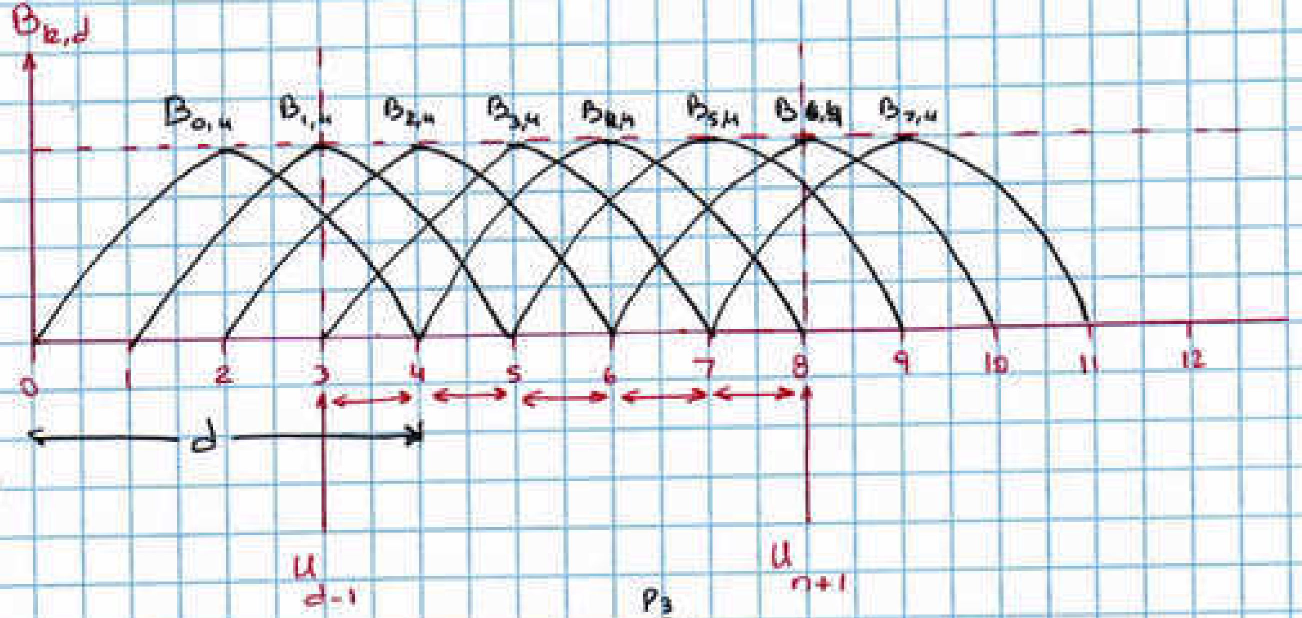
If $d - 1 = 3$ (3rd degree) then $\boxed{d = 4}$

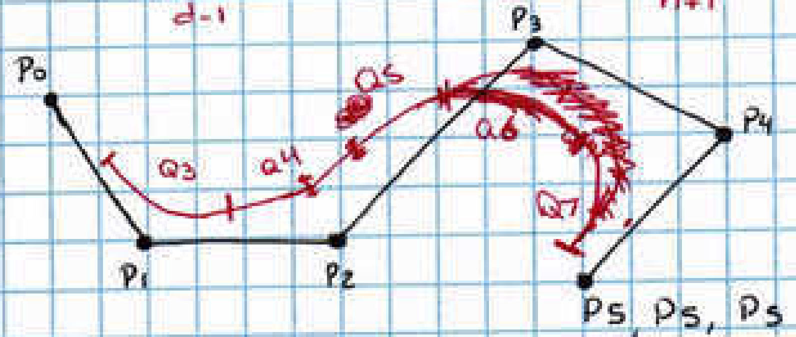
Number of knot values $= n + d + 1 = 7 + 4 + 1 = \boxed{12}$

Division of Intervals $= n + d = 7 + 4 = \boxed{11}$

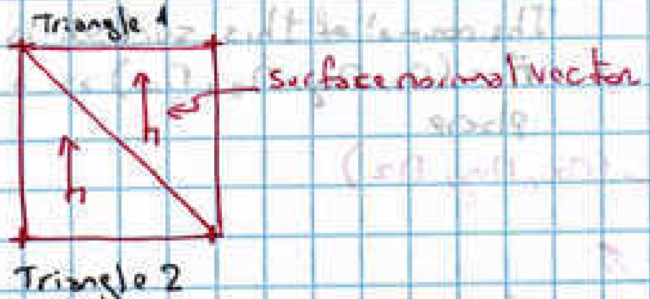
starts at $u_{d-1} = u_3$

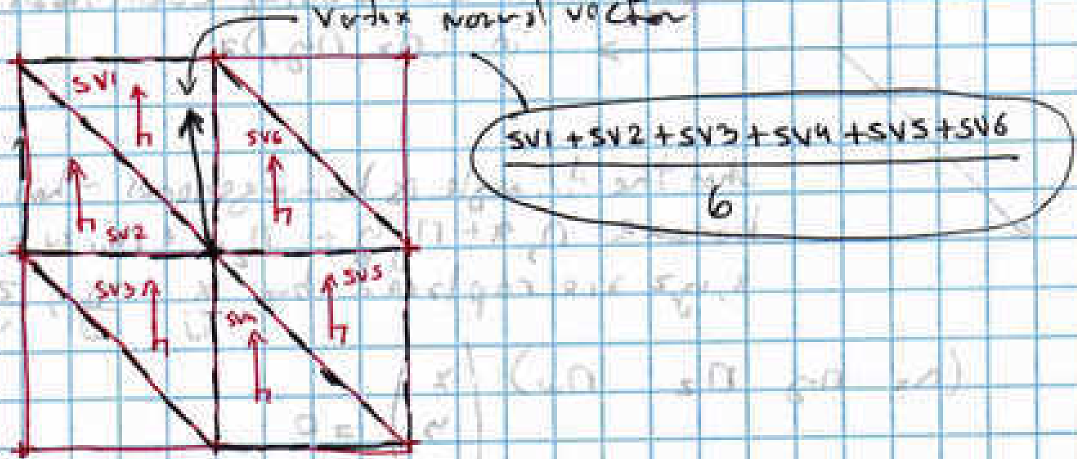
ends at $u_{n+1} = u_8$

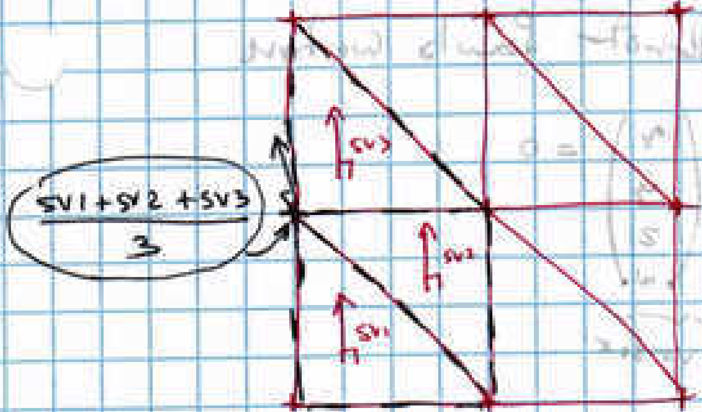


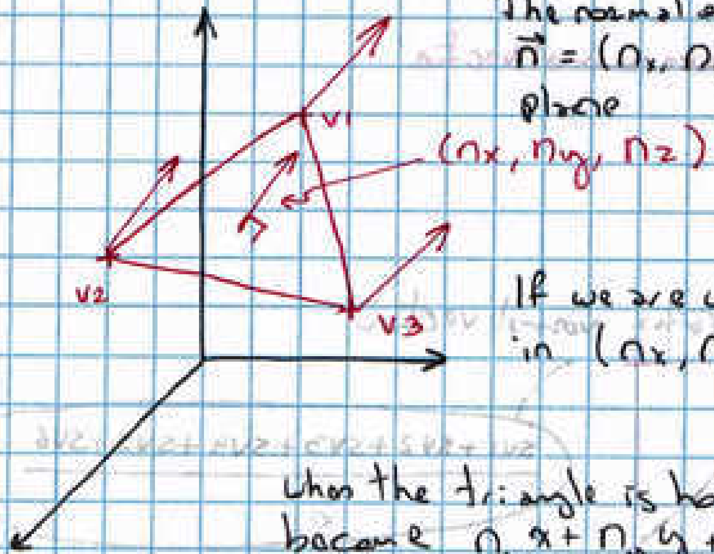


18B


Triangle 1
surface normal vector
n
n
Triangle 2


Vertex normal vector
SV1
SV2
SV3
SV4
SV5
SV6
n
$\frac{SV1 + SV2 + SV3 + SV4 + SV5 + SV6}{6}$


SV1
SV2
SV3
n
$\frac{SV1 + SV2 + SV3}{3}$



The normal of this surface is
$\vec{n} = (n_x, n_y, n_z, n_w)$ as a homogeneous plane

If we are using Euclidean space, the normal in (n_x, n_y, n_z)

When the triangle is homogeneous then the equation become $n_x x + n_y y + n_z z + n_w w = 0$ since x, y, z are replaced by $\frac{x}{w}, \frac{y}{w}, \frac{z}{w}$

$$(n_x \ n_y \ n_z \ n_w) \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = 0$$

then with the normal transformation facts we have

$$\underbrace{(n_x \ n_y \ n_z \ n_w) M^{-1}}_{\text{normal}} M \underbrace{\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}}_{\text{vertex}} = 0$$

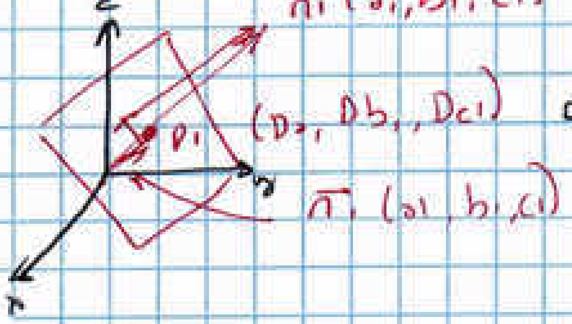
When we have the transformation using from the transforms vertex to the eye space, and the $n^T M^{-1}$ we obtain the normal vector in eye space

$$\begin{pmatrix} n_{x_{eye}} \\ n_{y_{eye}} \\ n_{z_{eye}} \\ n_{w_{eye}} \end{pmatrix} = (n_{x_{obj}} \ n_{y_{obj}} \ n_{z_{obj}} \ n_{w_{obj}}) M^{-1}$$

After converting pre multiplication to post-multiplication form we obtain

$$\begin{pmatrix} n_{x\,eye} \\ n_{y\,eye} \\ n_{z\,eye} \\ n_{w\,eye} \end{pmatrix} = (M^{-1})^T \begin{pmatrix} n_{x\,obj} \\ n_{y\,obj} \\ n_{z\,obj} \\ n_{w\,obj} \end{pmatrix}$$

The distance from Origin can be obtained by:



$$(a_1, b_1, c_1)\,(x - Da_1, y - Db_1, z - Dc_1) = 0$$

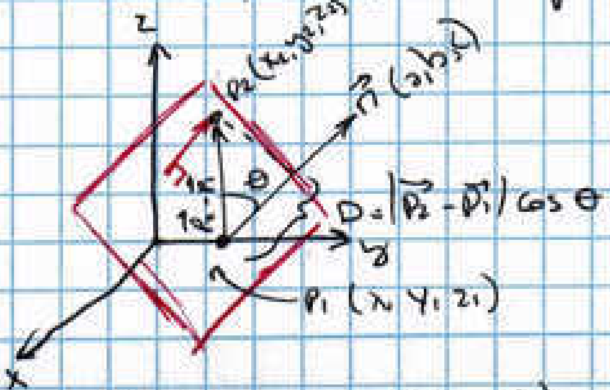
$$a_1(x - Da_1) + b_1(y - Db_1) + c_1(z - Dc_1) = 0$$

$$a_1x + b_1y + c_1z - D(a^2 + b^2 + c^2) = 0$$

$$a_1x + by + c_1z - D = 0$$

$$\sqrt{a_1^2 + b_1^2 + c_1^2} = 1$$

The distance from the point



$$D = |\vec{P_2} - \vec{P_1}| \cos\theta$$

$$= \frac{\vec{n}(\vec{P_2} - \vec{P_1})}{|\vec{n}|}$$

$$= \frac{\vec{n}(\vec{P_2} - \vec{P_1})}{\sqrt{a^2 + b^2 + c^2}}$$

therefore $\vec{n}(\vec{P_2} - \vec{P_1}) = |\vec{n}|\,|\vec{P_2} - \vec{P_1}| \cos\theta$

$$\vec{n} \cdot (\vec{P_2} - \vec{P_1}) = (a, b, c) \cdot (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

$$= a_{x_2} + b_{y_2} + c_{z_2} - (a_{x_1} + b_{y_1} + c_{z_1})$$

Then the distance formula, we have:

$$D = \frac{ax_2 + by_2 + cz_2 - (ax_1 + by_1 + cz_1)}{\sqrt{a^2 + b^2 + c^2}}$$

Diffuse Reflection

Using the Lambertian model in opengl, we can implement the diffuse reflection which occur in flat surface.

N: Surface Normal
L: light source
k: reflection coefficient
vertical component of light $(\vec{N} \cdot L)$

with k, we obtain the intensity of the surface $I = I_{source}\, k (\vec{N} \cdot \vec{L})$

Specular Reflection